Appendix E: Compatibility of Basic Properties

E.1 Pessimistic Confidentiality and Integrity

$$PC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(mod)\} \cdot \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x A$$
$$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(mod)\} \cdot A \in \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x A$$
$$PI \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(obs) \cup \Phi(A,mod)\} \cdot \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x A$$
$$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists B : \mathcal{F}ma\{\Phi(obs) \cup \Phi(A,mod)\} \cdot B \in \mathcal{F}ma\{\Phi(obs) \cup \Phi(A,mod)\} \wedge \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x B$$

Compatible Property Domain:

$$Q_{PC} * Q_{PI} = A,B : \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(mod)) \cap (\Phi(A,mod) \cup \Phi(obs)\}$$
$$= A,B : \mathcal{F}ma\{(\Phi(A) \cap \Phi(obs) \cup \Phi(mod)) \cap ((\Phi(A) \cap \Phi(mod) \cup \Phi(obs))\}$$
$$= A,B : \mathcal{F}ma\{(\Phi(A) \cap \Phi(obs)) \cap (\Phi(A) \cap \Phi(mod))\}$$
$$= A,B : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\}$$

If the sets of events for the system, $\Phi(obs)$ and $\Phi(mod)$ are defined such that $\Phi(obs) = \Phi(A,obs)$ and $\Phi(mod) = \Phi(A,mod)$ then the properties of pessimistic confidentiality and integrity will be compatible. Otherwise, the compatible property domain is incompatible with the individual property domains.

If the compatible property domain is compatible with the individual property domains, or the individual property domains can be made compatible, then the compatibility assessment continues with the compatible property specification.

Required World:

$$W_{PC} \wedge W_{PI} = (\Phi(A,obs) \cup \Phi(mod)) \cap (\Phi(A,mod) \cup \Phi(obs))\}$$
$$= \Phi(A,obs) \cup \Phi(A,mod)$$

Thus the assumptions about the events that are allowable in the composite environment must correspond the to required world.

Compatible Property Specification:

$$R_{PC} \wedge R_{PI} = \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x A \wedge \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x B$$
$$= \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x (A \wedge B)$$

Compatible Property:

$$PC \wedge PI = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A,B : \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(A,mod)\} \cdot A,B \in \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(A,mod)\} \wedge \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x (A \wedge B)$$
$$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(A,mod)\} \cdot A \in \mathcal{F}ma\{\Phi(A,obs) \cup \Phi(A,mod)\} \wedge \sigma(\mathbb{N}^{+}\mathbb{E};i)(in) \models_x A$$

E.2 Environment Optimistic Integrity and Pessimistic Integrity

$EOI \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \cdot$
$\exists B : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\} \cdot B \in \mathcal{F}ma\{\Phi(i,obs) \cup \Phi(i(A),mod)\}$
$\wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

Separating into sub-properties:

$EOI_1 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \cdot$
$A \in \mathcal{F}ma\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A$

$EOI_2 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists B : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$B \in \mathcal{F}ma\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{E};i)(in) \models_x B$

$PI \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)\} \cdot \sigma(\mathbb{N}+\mathbb{E};i)(in) \models_x A$

Separating into sub-properties and changing event variable names:

$PI_1 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists X : \mathcal{F}ma\{\Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)\} \cdot \sigma(\mathbb{N};i)(in) \models_x X$

$PI_2 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists \Delta : \mathcal{F}ma\{\Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)\} \cdot \sigma(\mathbb{E};i)(in) \models_x \Delta$

Compatible Property Domain:

$Q_{EOI_1} * Q_{PI} = A,X : \mathcal{F}ma\{\Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)\}$

$Q_{EOI_2} * Q_{PI} = B,X : \mathcal{F}ma\{\Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)\}$

Required World:

$W_{EOI_1} \wedge W_{PI} = A\ \Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)$

$W_{EOI_2} \wedge W_{PI} = \Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)$

Compatible Property Specification:

$R_{EOI_1} \wedge R_{EOI_2} \wedge R_{PI} = \sigma(\mathbb{N};i)(in) \models_x (A \wedge X) \wedge \sigma(\mathbb{E};i)(in) \models_x (B \wedge \Delta)$

Compatible Property:

$EOI \wedge PI = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A,B : \mathcal{F}ma\{\Phi(\mathbb{N},obs) \cup \Phi(\mathbb{N}(A),mod)\} \cdot$
$\wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$
$= PI$

E.3 Component Optimistic Confidentiality and Environment Optimistic Confidentiality

$COC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$\exists B : \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot$
$A,B \in \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

Separating into sub-properties:

$COC_1 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$A \in \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A$

$COC_2 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists B : \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot$
$B \in \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{E};i)(in) \models_x B$

$EOC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(i,mod)\} \cdot$

$\exists B : \mathit{Fma}\{\Phi(i,obs) \cup \Phi(mod)\} \cdot$

$B \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(i,mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

Separating into sub-properties:

$EOC_1 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists X : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot$

$X \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x X$

$EOC_2 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists \Delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\Delta \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{E};i)(in) \models_x \Delta$

Compatible Property:

$COC \wedge EOC = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A,B : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot$

$A,B \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

E.4 Component Optimistic Confidentiality and Strongly Optimistic Integrity

$COC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\exists B : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot$

$A,B \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

Separating into sub-properties:

$COC_1 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$A \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A$

$COC_2 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists B : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot$

$B \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{E};i)(in) \models_x B$

$SOI \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A,B : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\exists A,B \in \mathit{Fma}\{\Phi(obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

Separating into sub-properties and changing event variable names:

$SOI_1 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists X : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\exists X \in \mathit{Fma}\{\Phi(obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x X$

$SOI_2 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists \Delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\exists \Delta \in \mathit{Fma}\{\Phi(obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{E};i)(in) \models_x \Delta$

Compatible Property:

$COC \wedge SOI = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A,B : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$A,B \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(i(A),mod)\}$

$\wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

E.5 Authenticity and Component Optimistic Integrity

$AU \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathit{Fma}\{\Phi(i)\} \cdot \sigma\{\mathbf{i}\}(out) \models_x A$

$COI \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\exists B : \mathit{Fma}\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \cdot$

$A \in \mathit{Fma}\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

Separating into sub-properties:

$$Q_{COI_1} = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists X : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$$
$$X \in \mathit{Fma}\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x X$$

$$Q_{COI_2} = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists B : \mathit{Fma}\{\Phi(i,obs) \cup \Phi(mod)\} \cdot$$
$$B \in \mathit{Fma}\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \wedge \sigma(\mathbb{E};i)(in) \models_x B$$

Compatible Property:

$$AU \wedge COI = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists X : \mathit{Fma}\{\Phi(i,obs) \cup \Phi(i,mod)\} \cdot$$
$$\exists A,B : \mathit{Fma}\{\Phi(i,obs) \cup \Phi(i(A),mod)\} \cdot$$
$$A,B,X \in \mathit{Fma}\{\Phi(i,obs) \cup \Phi(i(A),mod)\}$$
$$\wedge \sigma(i;\mathbb{N}+\mathbb{E})(out) \models_x A \wedge \sigma(\mathbb{N};i)(in) \models_x X \wedge \sigma(\mathbb{E};i)(in) \models_x B$$

E.6 Accessibility and Strongly Optimistic Confidentiality

$$AC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \Phi \cdot \sigma\{\mathbf{i}\}(in) \models_x \alpha \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond \alpha$$

$$SOC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A,B : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$$
$$A,B \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(i,mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$$

Compatible Property:

$$AC \wedge SOC$$
$$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$$
$$\exists \beta,\chi : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot \alpha,\beta,\chi \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(i,mod)\}$$
$$\wedge (\sigma(\mathbb{N}+\mathbb{E};i)(in) \models_x \alpha \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond \alpha) \wedge \sigma(\mathbb{N};i)(in) \models_x \chi \wedge \sigma(\mathbb{E};i)(in) \models_x \beta$$

$$Fix\ \alpha = \beta$$
$$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$$
$$\exists \beta,\chi : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot \alpha,\beta,\chi \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(i,mod)\}$$
$$\wedge (\sigma(\mathbb{N}+\mathbb{E};i)(in) \models_x \alpha \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond \alpha) \wedge \sigma(\mathbb{N};i)(in) \models_x \chi \wedge \sigma(\mathbb{E};i)(in) \models_x \alpha$$

$$Fix\ \alpha = \chi$$
$$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$$
$$\exists \beta,\chi : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot \alpha,\beta,\chi \in \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(i,mod)\}$$
$$\wedge (\sigma(\mathbb{N}+\mathbb{E};i)(in) \models_x \alpha \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond \alpha) \wedge \sigma(\mathbb{N};i)(in) \models_x \alpha \wedge \sigma(\mathbb{E};i)(in) \models_x \beta$$

E.7 Eventual Progress and Accessibility

$$EP \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \Phi \cdot \exists \chi : \Phi \cdot \sigma\{\mathbf{i}\} \models_x \Diamond \alpha \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond \chi$$

$$AC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \beta : \Phi \cdot \sigma\{\mathbf{i}\}(in)[\ell_i] \models_x \beta \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond \beta$$

Compatible Property:

EP $\wedge$ AC

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha,\beta : \Phi \cdot \exists \chi : \Phi \cdot$

$(\sigma\{\mathbf{i}\} \models_x \Diamond\alpha \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond\chi) \wedge (\sigma\{\mathbf{i}\}(in) \models_x \beta \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond\beta)$

Fix $\alpha = \beta$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha,\beta : \Phi \cdot \exists \chi : \Phi \cdot$

$(\sigma\{\mathbf{i}\} \models_x \Diamond\beta \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond\chi) \wedge (\sigma\{\mathbf{i}\}(in) \models_x \beta \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond\beta)$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha,\beta : \Phi \cdot \exists \chi : \Phi \cdot$

$\sigma\{\mathbf{i}\}(in) \models_x \beta \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond\chi$

Fix $\chi = \beta$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha,\beta : \Phi \cdot \exists \chi : \Phi \cdot$

$(\sigma\{\mathbf{i}\} \models_x \Diamond\alpha \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond\beta) \wedge (\sigma\{\mathbf{i}\}(in) \models_x \beta \Rightarrow \sigma\{\mathbf{i}\} \models_x \Diamond\beta)$

E.8 Environment Optimistic Confidentiality and Faithfulness

$EOC \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(i,mod)\} \cdot$

$\exists B : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$B \in \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(i,mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x A \wedge \sigma(\mathbb{E};i)(in) \models_x B$

Separating into sub-properties:

$EOC_1 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists X : \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot$

$X \in \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{N};i)(in) \models_x X$

$EOC_2 \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists \Delta : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\Delta \in \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \wedge \sigma(\mathbb{E};i)(in) \models_x \Delta$

$FT \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall\forall \alpha : \mathcal{F}ma\{\Phi\} \cdot \exists \beta : \mathcal{F}ma\{\Phi\} \cdot$

$((\alpha;\beta) \wedge \sigma\{\mathbf{i}\}(in) \models_x \alpha) \Rightarrow \sigma\{\mathbf{i}\}(out) \models_x \Diamond\beta$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathcal{F}ma\{\Phi\} \cdot \exists \beta : \mathcal{F}ma\{\Phi\} \cdot$

$((\alpha;\beta) \wedge \sigma(\mathbb{N}+\mathbb{E};i)(in) \models_x \alpha) \Rightarrow \sigma(i;\mathbb{N}+\mathbb{E})(out) \models_x \Diamond\beta$

Compatible Property:

EOC $\wedge$ FT

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$

$\exists \beta : \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$

$\exists \chi : \mathcal{F}ma\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot \exists \delta : \mathcal{F}ma\{\Phi(obs) \cup \Phi(mod)\} \cdot$

$\alpha,\beta,\chi \in \mathcal{F}ma\{ (\Phi(i(A),obs) \cup \Phi(mod)) \}$

$\wedge \Delta \in \mathcal{F}ma\{ (\Phi(i(A),obs) \cup \Phi(mod)) \}$

$\wedge (((\alpha;\beta) \wedge (\sigma(\mathbb{N};i)(in) \models_x \alpha \vee \sigma(\mathbb{E};i)(in) \models_x \alpha))$

$\Rightarrow (\sigma(i;\mathbb{N})(out) \models_x \Diamond\beta \vee \sigma(i;\mathbb{E})(out) \models_x \Diamond\beta))$

$\wedge \sigma(\mathbb{N};i)(in) \models_x \chi \wedge \sigma(\mathbb{E};i)(in) \models_x \delta$

Fix $\alpha = \chi$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \beta : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \chi : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot \exists \delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$\alpha,\beta,\chi \in \mathit{Fma}\{ (\Phi(i(A),obs) \cup \Phi(mod)) \}$
$\wedge \Delta \in \mathit{Fma}\{ (\Phi(i(A),obs) \cup \Phi(mod)) \}$
$\wedge (((\alpha;\beta) \wedge (\sigma(\mathbb{N};i)(in)[\not{k}_i] \models_x \alpha \vee \sigma(\mathbb{E};i)(in)[\not{k}_i] \models_x \alpha))$
$\Rightarrow (\sigma(i;\mathbb{N})(out) \models_x \Diamond\beta \vee \sigma(i;\mathbb{E})(out) \models_x \Diamond\beta))$
$\wedge \sigma(\mathbb{N};i)(in) \models_x \alpha \wedge \sigma(\mathbb{E};i)(in) \models_x \delta$

Fix $\beta = \chi$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \beta : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \chi : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot \exists \delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$\alpha,\beta,\chi \in \mathit{Fma}\{(\Phi(i(A),obs) \cup \Phi(mod)) \}$
$\wedge \Delta \in \mathit{Fma}\{ (\Phi(i(A),obs) \cup \Phi(mod)) \}$
$\wedge (((\alpha;\beta) \wedge (\sigma(\mathbb{N};i)(in) \models_x \alpha \vee \sigma(\mathbb{E};i)(in) \models_x \alpha))$
$\Rightarrow (\sigma(i;\mathbb{N})(out) \models_x \Diamond\beta \vee \sigma(i;\mathbb{E})(out) \models_x \Diamond\beta))$
$\wedge \sigma(\mathbb{N};i)(in) \models_x \beta \wedge \sigma(\mathbb{E};i)(in) \models_x \delta$

Fix $\alpha = \delta$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \beta : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \chi : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot \exists \delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$\alpha,\beta,\chi \in \mathit{Fma}\{ (\Phi(i(A),obs) \cup \Phi(mod)) \} \wedge \Delta \in \mathit{Fma}\{ \Phi(i(A),obs) \cup \Phi(mod) \}$
$\wedge (((\alpha;\beta) \wedge (\sigma(\mathbb{N};i)(in) \models_x \alpha \vee \sigma(\mathbb{E};i)(in) \models_x \alpha))$
$\Rightarrow (\sigma(i;\mathbb{N})(out) \models_x \Diamond\beta \vee \sigma(i;\mathbb{E})(out) \models_x \Diamond\beta))$
$\wedge \sigma(\mathbb{N};i)(in) \models_x \chi \wedge \sigma(\mathbb{E};i)(in) \models_x \alpha$

Fix $\beta = \delta$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \beta : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \chi : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot \exists \delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$\alpha,\beta,\chi \in \mathit{Fma}\{ (\Phi(i(A),obs) \cup \Phi(mod)) \}$
$\wedge \Delta \in \mathit{Fma}\{ (\Phi(i(A),obs) \cup \Phi(mod)) \}$
$\wedge (((\alpha;\beta) \wedge (\sigma(\mathbb{N};i)(in) \models_x \alpha \vee \sigma(\mathbb{E};i)(in) \models_x \alpha))$
$\Rightarrow (\sigma(i;\mathbb{N})(out) \models_x \Diamond\beta \vee \sigma(i;\mathbb{E})(out) \models_x \Diamond\beta))$
$\wedge \sigma(\mathbb{N};i)(in) \models_x \chi \wedge \sigma(\mathbb{E};i)(in) \models_x \beta$

Fix $\alpha = \chi, \beta = \delta$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \beta : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \chi : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot \exists \delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$\alpha,\beta,\chi \in \mathit{Fma}\{(\Phi(i(A),obs) \cup \Phi(mod))\}$
$\wedge \Delta \in \mathit{Fma}\{(\Phi(i(A),obs) \cup \Phi(mod))\}$
$\wedge (((\alpha;\beta) \wedge (\sigma(\mathbb{N};i)(in) \models_x \alpha \vee \sigma(\mathbb{E};i)(in) \models_x \alpha))$
$\Rightarrow (\sigma(i;\mathbb{N})(out) \models_x \Diamond\beta \vee \sigma(i;\mathbb{E})(out) \models_x \Diamond\beta))$
$\wedge \sigma(\mathbb{N};i)(in) \models_x \alpha \wedge \sigma(\mathbb{E};i)(in) \models_x \beta$

Fix $\alpha = \delta, \beta = \chi$

$= \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \beta : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod))\} \cdot$
$\exists \chi : \mathit{Fma}\{\Phi(i(A),obs) \cup \Phi(mod)\} \cdot \exists \delta : \mathit{Fma}\{\Phi(obs) \cup \Phi(mod)\} \cdot$
$\alpha,\beta,\chi \in \mathit{Fma}\{(\Phi(i(A),obs) \cup \Phi(mod))\}$
$\wedge \Delta \in \mathit{Fma}\{(\Phi(i(A),obs) \cup \Phi(mod))\}$
$\wedge (\alpha=\delta) \wedge (\beta=\chi) \wedge (((\alpha;\beta) \wedge (\sigma(\mathbb{N};i)(in) \models_x \delta \vee \sigma(\mathbb{E};i)(in) \models_x \delta))$
$\Rightarrow (\sigma(i;\mathbb{N})(out) \models_x \Diamond\beta \vee \sigma(i;\mathbb{E})(out) \models_x \Diamond\beta))$
$\wedge \sigma(\mathbb{N};i)(in) \models_x \beta \wedge \sigma(\mathbb{E};i)(in) \models_x \delta$

E.9 Faithfulness and Eventual Output

$FT \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \Phi \cdot \exists \beta : \Phi \cdot$
$((\alpha;\beta) \wedge \sigma\{\mathbf{i}\}(in) \models_x \alpha) \Rightarrow \sigma\{\mathbf{i}\}(out) \models_x \Diamond\beta$

$EO \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall \alpha : \Phi \cdot \exists \beta : \Phi \cdot \sigma\{\mathbf{i}\} \models_x \alpha \Rightarrow \sigma\{\mathbf{i}\}(out) \models_x \Diamond\beta$

Compatible Property:

$FT \wedge EO = \forall \alpha \cdot \exists \beta : \Phi \cdot ((\alpha;\beta) \wedge \sigma\{\mathbf{i}\}(in) \models_x \alpha) \Rightarrow \sigma\{\mathbf{i}\}(out) \models_x \Diamond\beta$

E.10 Authenticity and Provenance

$AU \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \exists A : \mathit{Fma}\{\Phi^i(A)\} \cdot \sigma(i^+\mathbb{E};\mathbb{N}^+\mathbb{E})(out) \models_x A$

$PV \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall A : \mathit{Fma}\{\Phi\} \cdot \sigma\{\mathbf{i}\}(in) \models_x \Diamond A \Rightarrow \sigma(\mathbb{N}^+\mathbb{E};i)(out) \models_x A$
$\forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall B : \mathit{Fma}\{\Phi\} \cdot \sigma\{\mathbf{i}\}(in) \models_x \Diamond B \Rightarrow \sigma(\mathbb{N}^+\mathbb{E};i)(out) \models_x B$

Compatible Property:

$AU \wedge PV = \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall B \cdot \exists A : \mathit{Fma}\{\Phi(i(A))\} \cdot$
$(\sigma(i^+\mathbb{E};\mathbb{N}^+\mathbb{E})(out) \models_x A) \wedge (\sigma\{\mathbf{i}\}(in) \models_x \Diamond B \Rightarrow \sigma(\mathbb{N}^+\mathbb{E};i)(out) \models_x B)$

E.11 Provenance and Utility

$PV \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall A : \mathcal{F}ma\{\Phi\} \cdot \sigma\{\mathbf{i}\}(in) \models_x \Diamond A \Rightarrow \sigma(\mathbb{N}^+ \mathbb{E};i)(out) \models_x A$

$UT \equiv \forall x : \mathbb{N} \cdot \forall i : \mathbb{N} \cdot \forall A : \mathcal{F}ma\{\Phi\} \cdot \sigma(\mathbb{N}^+ \mathbb{E};i)(out) \models_x A \Rightarrow \sigma\{\mathbf{i}\}(in) \models_x \Diamond A$

Compatible Property:

$PV \wedge UT = \forall A : \mathcal{F}ma\{\Phi(i(A))\} \cdot \sigma\{\mathbf{i}\}(in) \models_x \Diamond A \wedge \sigma(\mathbb{N}^+ \mathbb{E};i)(out) \models_x A$