Light and Reflectance Models

We seek image understanding:

> **Understand:** To have an internal model which reflects the significant structure and behaviour of the entity (cf. COD).

Therefore we need models of

- Light
- Reflectance
- Cameras/eyes
- Scenes, actions, and events

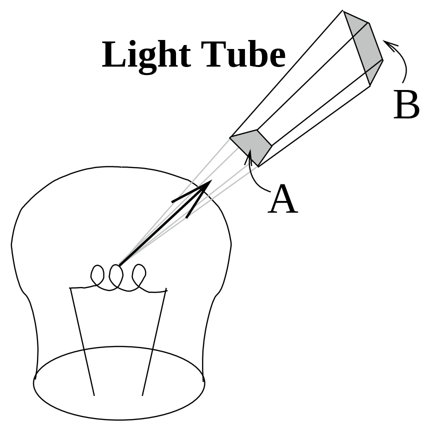
Computer graphics require similar models.

Here we describe some basic models for light and reflectance. In a subsequent section we will describe camera models.

Light Tube Analogy

Assume a non-absorbing, non-scattering, uniform medium, such as a vacuum or (apprx) air. Then geometrical optics models light as travelling along straight lines called rays.

Ignore time-delays due to the finite speed of light, and ignore the diffraction of light.

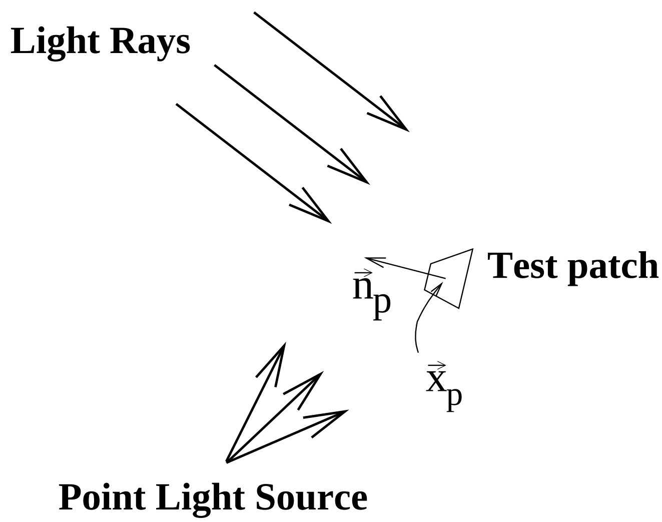


Given a collection of rays that lie within a tube and which cross both ends of the tube (i.e. A and B), but do not cross the sides of the tube, **then** the power of the light impinging on end A, due to these rays, is equal to the power impinging on B (due to the same rays).

Measurement of Light: Irradiance

Irradiance describes the power of the light arriving at a (possibly virtual) test patch.

In particular, $I(\lambda, \vec{x}_p, \vec{n}_p)$ is the power, per unit wavelength and per unit surface area, of light impinging on a small patch at position $\vec{x}_p$, with surface normal $\vec{n}_p$:



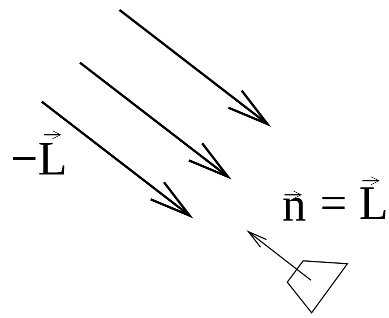
Units: $I(\lambda, \vec{x}_p, \vec{n}_p)$ is in $\frac{W}{(nm)m^2}$, which is Watts per unit wavelength (i.e. nm), per unit surface area of the test patch. *Caution:* Note the dependence on the patch orientation (i.e. $\vec{n}_p$).

Visible Spectrum: Wavelengths λ between $400nm$ and $700nm$ (see the Matlab colour tutorial). Due to the dependence on λ, $I(\lambda, \vec{x}_p, \vec{n}_p)$ is referred to as an *irradiance spectral density*.

Light Sources

1. **Distant Compact Source**.

   Suppose the source is far away and has a small angular extent (eg. the sun). Then the light rays are (approximately) parallel, and the irradiance is independent of position $\vec{x}_p$.

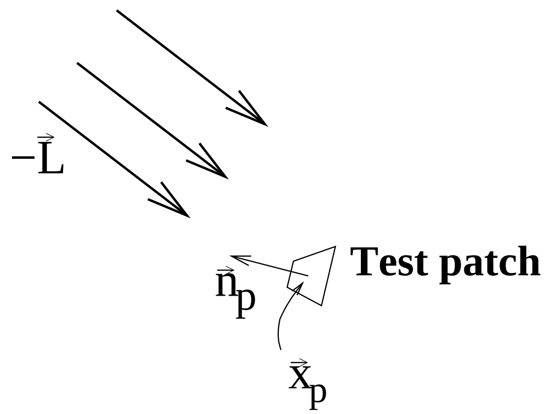


   Light Source Direction: Define $\vec{L}$ to be the unit vector in the direction of the light source as seen by the test patch.

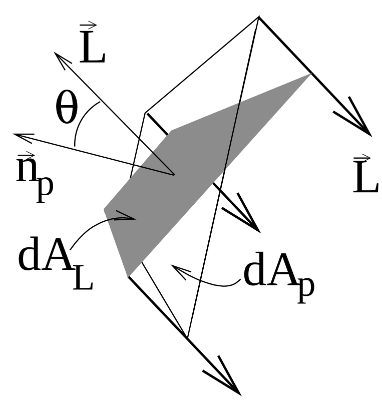
   Light Source Irradiance: Use a patch with normal $\vec{n} = \vec{L}$ to define the irradiance for a distant light source (this is the orientation which catches the most light). Given the invariance over the test patch position, we can drop the $\vec{x}_p$ from the light source irradiance $I(\lambda, \vec{x}_p, \vec{L})$ and write $I(\lambda, \vec{L})$ instead.

Distant Compact Source (Cont.)

Surface Irradiance. What's the irradiance impinging on a small patch of area dA_p in a general orientation (i.e. with surface normal $\vec{n}_p \neq \vec{L}$)?

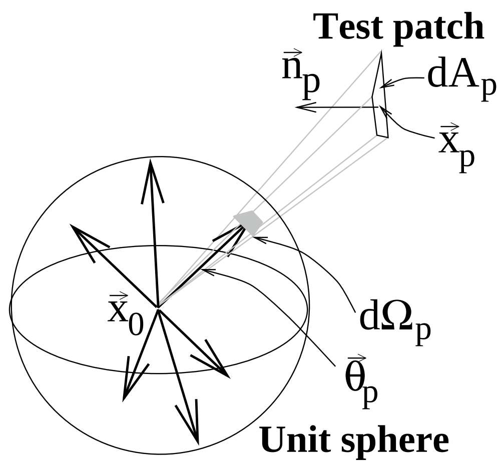


Projected Area. The power impinging on the patch is just the power passing through the projected area of the patch, where the projection is perpendicular to the direction of the light rays. The projected area is $dA_L = |\cos(\theta)| dA_p = |\vec{n}_p \cdot \vec{L}| dA_p$.



For an opaque surface we require $\vec{n}_p \cdot \vec{L} >= 0$, since otherwise it is in shadow. Therefore, the surface irradiance is $I(\lambda, \vec{n}_p) = \lfloor \vec{n}_p \cdot \vec{L} \rfloor I(\lambda, \vec{L})$. Here $\lfloor x \rfloor$ denotes the rectification of x, namely x when $x > 0$ and 0 otherwise.

2. **Point Light Source**



The **radiant intensity** $R(\lambda, \vec{\theta}_p)$ of a point light source is the power, per unit solid angle, per unit wavelength λ, impinging on a patch situated in the direction $\vec{\theta}_p$.

Solid angle is defined to be the area $d\Omega_p$ of the projection onto the unit sphere. For a small patch,

$$d\Omega_p = \frac{|\vec{n}_p \cdot \vec{L}|}{||\vec{x}_p - \vec{x}_0||^2}\, dA_p,$$

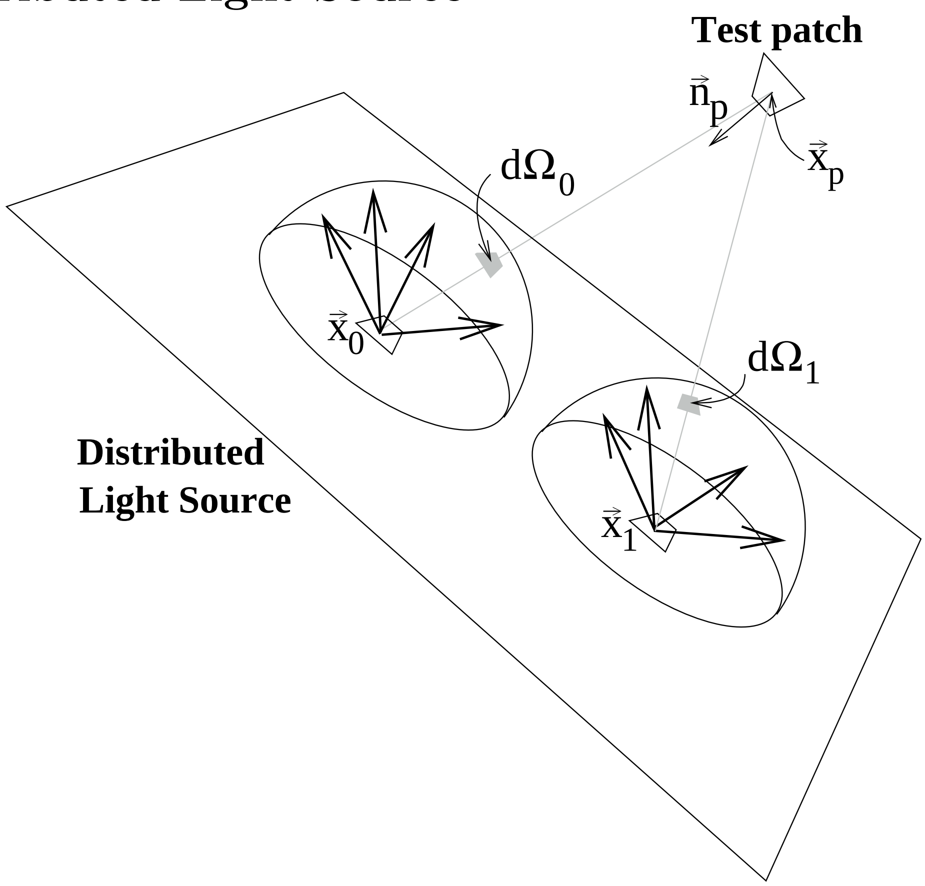
where $\vec{L} = (\vec{x}_0 - \vec{x}_p)/||\vec{x}_0 - \vec{x}_p||$.

Units: $R(\lambda, \vec{\theta})$ is in Watts per unit wavelength, per unit solid angle (i.e. steradian, aka sr), that is $\frac{W}{(nm)(sr)}$.

Computation of Surface Irradiance: For the test patch above, the surface irradiance is

$$I(\lambda, \vec{x}_p, \vec{n}_p) = \left[R(\lambda, \vec{\theta}_p) d\Omega_p\right]/dA_p = \frac{\lfloor \vec{n}_p \cdot \vec{L} \rfloor}{||\vec{x}_p - \vec{x}_0||^2}\, R(\lambda, \vec{\theta}_p).$$

3. **Distributed Light Source**



Radiance: For a distributed light source the radiance $R(\lambda, \vec{x}_s, \vec{\theta})$ is parameterized by the spatial position $\vec{x}_s$ within the light source. It is in units of $\frac{W}{(nm)m^2 sr}$, where the area measure, m^2, denotes surface area on the light source.

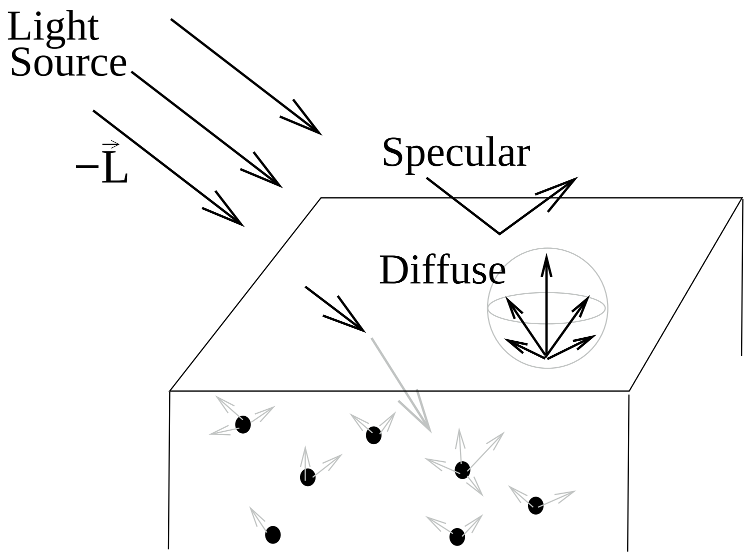
Irradiance on a Test Patch: The irradiance impinging on a test patch is obtained by integrating the point-source contributions of infinitesimal patches over the light source, namely terms of the form

$$\frac{\lfloor \vec{n}_p \cdot \vec{L}(\vec{x}, \vec{x}_p) \rfloor}{||\vec{x} - \vec{x}_p||^2} R(\lambda, \vec{x}, \vec{\theta}(\vec{x}_p, \vec{x})) d\vec{x}.$$

Moral: Combining several lights is **linear**; the contributions to the surface irradiance can simply be added (integrated).

Models of Reflectance

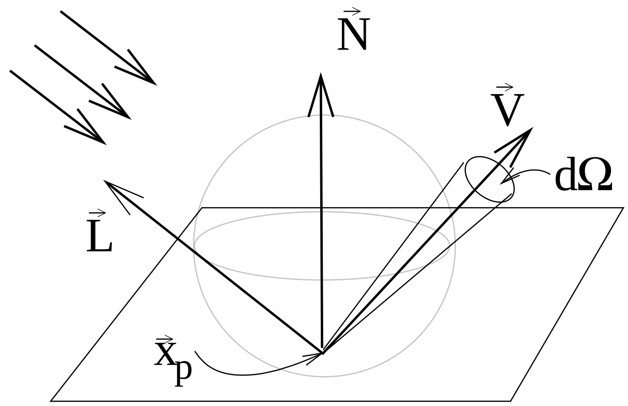
Two types of reflectance:



- **Specular Reflectance**: Reflectance from the surface, primarily in the "mirror reflection" direction. The spectral distribution of the reflected light can be the same as the incident light.
- **Diffuse Reflectance**: Light is absorbed and re-emitted from the body, scattering in all directions. The spectral distribution of the reflected light depends on the pigmentation of the object.

Diffuse Reflectance

Suppose the surface irradiance is $I(\lambda, \vec{N})$, say coming from a distant compact source in the direction $\vec{L}$.



Here $\vec{N}$ is the surface normal, and $\vec{V}$ is the viewer direction.

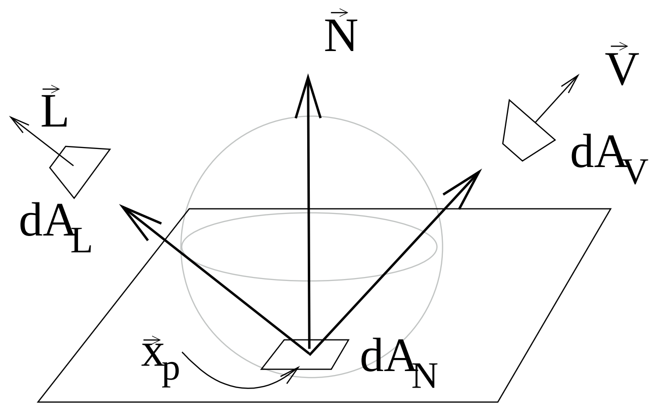
The **reflected radiance** (per unit surface area) is given by:

$$R(\lambda, \vec{x}_p, \vec{V}; \vec{N}) = r(\lambda, \vec{L}, \vec{V}) \lfloor \vec{N} \cdot \vec{V} \rfloor I(\lambda, \vec{N})$$

The *bidirectional reflectance distribution function* (BRDF), $r(\lambda, \vec{L}, \vec{V})$, gives the proportion of the incident light at wavelength λ scattered in the direction $\vec{V}$. (Note: Here $\vec{L}$ and $\vec{V}$ are unit vectors defined in the surface coordinate frame.)

Lambertian Approximation: The BRDF is assumed to be invariant of both $\vec{L}$ and $\vec{V}$, that is $r(\lambda, \vec{L}, \vec{V}) = r(\lambda)$.

Lambertian Reflectance: Standard Units



Recall that the light source irradiance (per unit area perpendicular to $\vec{L}$) is related to the surface irradiance by:

$$I(\lambda, \vec{N}) = \lfloor \vec{N} \cdot \vec{L} \rfloor I(\lambda, \vec{L})$$

Similarly, the conversion from cross-sectional area perpendicular to $\vec{V}$, say dA_V, to the corresponding area on the reflecting surface, dA_N, is

$$dA_V = |\vec{N} \cdot \vec{V}| dA_N$$

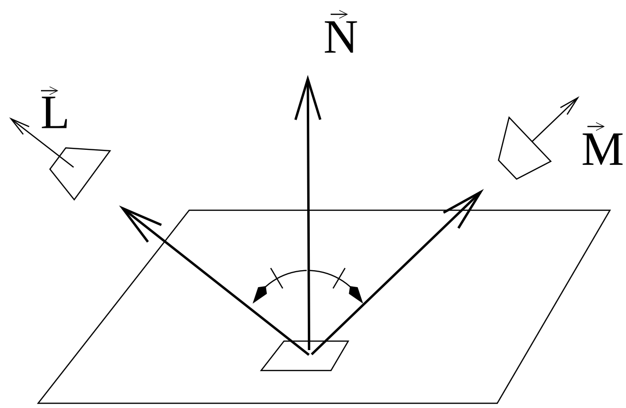
It follows that the radiance *per unit area perpendicular to* $\vec{V}$, due to diffuse reflection from a Lambertian surface, is given by

$$\boxed{R(\lambda, \vec{x}_p, \vec{V}) = r(\lambda) \lfloor \vec{N} \cdot \vec{L} \rfloor I(\lambda, \vec{L})}$$

Note that this does not depend on the viewing direction $\vec{V}$.

Specular Reflectance

Consider a distant compact source with irradiance $I^i(\lambda, \vec{L})$. Here the superscript 'i' denotes the 'incident' irradiance.



Here $\vec{N}$ is the surface normal, $\vec{L}$ is the light-source direction, and $\vec{M}$ is the mirror reflection direction. These directions are related by

$$\vec{M} = -\vec{L} + 2\vec{N}\left[\vec{N} \cdot \vec{L}\right].$$

The reflected irradiance $I^r(\lambda, \vec{M})$ is given by

$$I^r(\lambda, \vec{M}) = F(\lambda, \vec{L}) I^i(\lambda, \vec{L}).$$

Here the *Fresnel term*, $F(\lambda, \vec{L})$, dictates any change in the spectral distribution.

Scattering of the specular reflection around the mirror reflection direction, $\vec{M}$, can also be modelled (see the Phong model below).

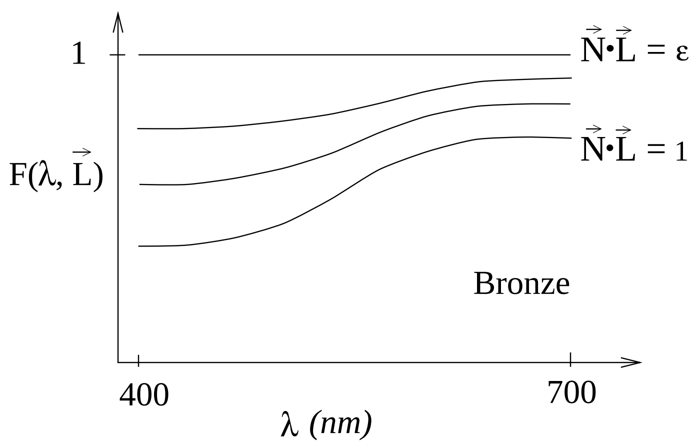
The 'Colour' of Highlights

For specular reflection the reflected irradiance $I^r(\lambda, \vec{M})$ is given by

$$I^r(\lambda, \vec{M}) = F(\lambda, \vec{L}) I^i(\lambda, \vec{L}).$$

For plastics, $F(\lambda, \vec{L}) \approx f(\vec{L} \cdot \vec{N})$, so there is little spectral change between the incident and reflected light.

However, metals do show spectral changes. Moreover these changes depend on the incident angle. For example, for bronze:



Ref: Cook and Torrance, A reflectance model for computer graphics, ACM Trans. on Graphics, 1(1), Jan. 1982, pp. 7-24.

Phong Radiance Model

The Phong shading model, commonly used in computer graphics, uses an approximation to the reflected radiance of a surface. Suppose the illumination is in the direction $\vec{L}$, with irradiance $I(\lambda) = I(\lambda, \vec{L})$. Let $\vec{N}$ be the surface normal, and $\vec{M}$ be the mirror reflection direction (as before).

Then the reflected radiance used by the Phong shading model at a surface point $\vec{x}_p$, per unit area perpendicular to the viewing direction $\vec{V}$, is

$$\boxed{R(\lambda, \vec{x}_p, \vec{V}) = k_a r(\lambda) + k_d r(\lambda) \lfloor \vec{N} \cdot \vec{L} \rfloor I(\lambda) + k_s S(\lambda) (\vec{M} \cdot \vec{V})^{k_e}}$$

Here

- $r(\lambda)$ is the diffuse spectral reflectance distribution for the surface;
- k_a, k_d, k_s are non-negative coefficients for the ambient, diffuse, and specular reflection terms, respectively;
- k_e is the spectral exponent, controlling the spread of the specular reflection (rougher surfaces modelled by smaller k_e);
- $S(\lambda)$ is the spectral distribution of the specular reflection. It is just $I(\lambda)$ for painted or plastic surfaces. For metals it can be approximated by some linear combination of $I(\lambda)$ and $r(\lambda)$.

See the Matlab file `phongDemo.m` to simulate the effects of varying these parameters.

See the Light

Our visual systems provide us with an interpretation of the scene.

It takes practice to actually pay attention to the *stimulus*, instead. That is, we may wish to "see the light" as opposed to the scene. (Note the reflectance illusions from the previous lecture illustrate some of the difficulties we might have.)

Once you can see the light, consider perception:

- How do we identify light sources (they aren't always the brightest objects in the image)?
- How do we identify highlights? Transparency? Haze?
- How do we identify metallic surfaces? Wax? Pearls?
- How do we perceive the pigmentation of a surface, $r(\lambda)$, separately from the illuminant $I(\lambda, \vec{L})$ and surface orientation $\vec{N}$, given only the reflected radiance R? Naively, this looks like, "Find r, I, and $\vec{N}$ given only the reflected radiance $R = r \times I \times \lfloor \vec{N} \cdot \vec{L} \rfloor$." (See p. 10.)
- ... and so on.